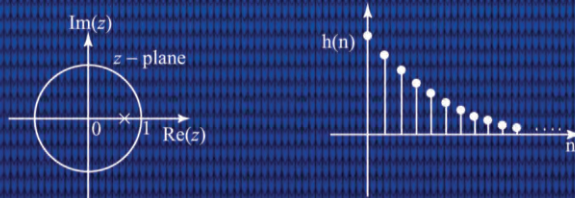


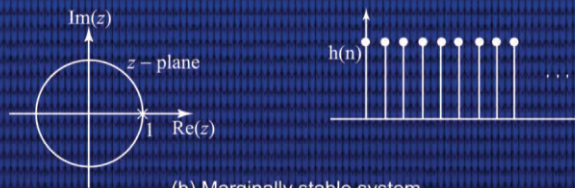


# Signals and Stochastic Process

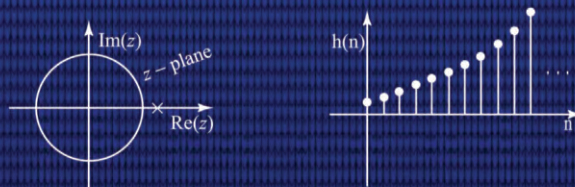
P. Ramesh Babu  
R. Anandanatarajan



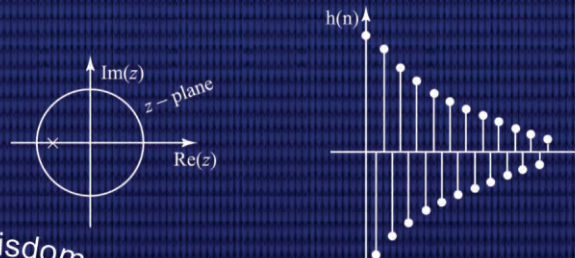
(a) Stable system



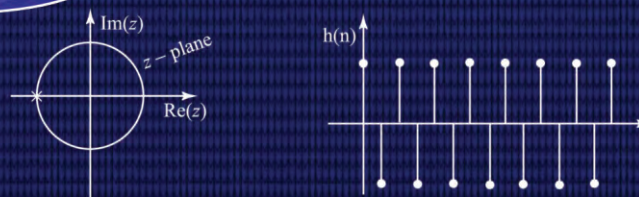
(b) Marginally stable system



(c) Unstable system



(d) Stable system



(e) Marginally stable system

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# Signals and Stochastic Process

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# Signals and Stochastic Process

This book is designed to meet the syllabus requirements of the Undergraduate courses of all the branches of Engineering. The contents of the book are presented in a lucid style so that even an average student can grasp the subject. More number of simple and complex problems are worked out to strengthen the theory.

## Salient features:

- Most of the topics are presented in a lucid manner so that the students of various branches can easily understand the subject.
- More than 445 solved problems, 235 practice problems and 120 multiple choice questions are given.
- Covers the entire syllabus of **JNTU**.
- Notations used in the book are identical to notations used in most of the DSP books. It is an ideal Introductory Textbook to Digital Signal Processing.
- MATLAB programs are included.

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